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A QUANTITATIVE APPROACH TO AGGREGATION

IN THE MODELING OF

TACTICAL COMMAND, CONTROL, AND COMMUNICATION SYSTEMS

by

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ABSTRACT

This research effort is directed at determining the feasibility of using structural-sensitivity measures as the basis for simplifying large mathematical models of dynamic systems (e.g., models of tactical command, control, and communication systems). Toward this end, a FORTRAN program was written which can be used to simplify models characterized by sets of linear differential equations. Specifically, the program determines the optimal simplified model (i.e., the set of coefficients characterizing the set of linear differential equations) of specified dimension corresponding to the original linear model of higher dimension.

The algorithm, on which the FORTRAN program is based, minimizes an objective function which is defined in terms of the structural sensitivities of the state variables to be preserved in the simplified model. The program inputs are the set of coefficients which define (1) the linear differential equations representing the original model, (2) the dimension of the simplified model to be determined, and (3) the set of parameters defining the objective function to be minimized. The program outputs are the set of coefficients which define the optimal simplified model and the value of the objective function corresponding to the optimal simplified model.

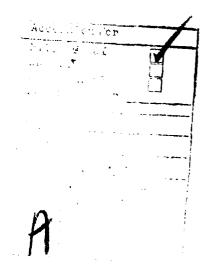... ... ... (...).

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INTRODUCTION

An approach is proposed to simplifying complex mathematical models of dynamic systems. The approach is based on the concept of structural sensitivity.

The mathematical model to be simplified is represented as a graph in which each link of the graph represents the dependency of one system variable on another. The state variables of the system are selected so as to include all the system variables of interest which are also to be included in the simplified model. The remaining state variables are selected so as to minimize the sensitivity of the interesting state variables to the cutting of all the links between the interesting state variables and the remaining state variables. When this minimization is realized, the optimal simplified model, which includes all the interesting state variables, can be "cut" out of the original system model. This process involves finding that transformation of the original state variables which preserves the interesting state variables while minimizing the structural sensitivity of these state variables with respect to the remaining state variables.

In situations where the best possible simplified model that can be cut out of the original system model is not an acceptable representation of the original system, the dimension of the simplified model can be increased to improve its accur-

acy. In this case, a transformation must be found which not
only preserves the interesting state variables, but which also
partitions the remaining state variables into two sets. One
set of state variables from this partition is added to the set
of interesting state variables to produce an augmented set of
interesting state variables; the simplified model will now in-
clude this agumented set and thus be of higher dimension, and
of improved accuracy. The other set of state variables from
the partition is selected so as to minimize the structural
sensitivity of the set of interesting state variables with
respect to this remaining set.

MODEL SIMPLIFICATION: THE GENERAL AGGREGATION PROBLEM

The general aggregation problem is stated here for the case that the system of interest is well modeled by a set of ordinary differential equations. However, the extention of these results to those important systems which are best modeled by discrete-event models is not trivial, particularly with respect to the computation of sensitivities within dynamic systems. Additional research is called for here.

Consider that the system to be studied is well modeled by a set of ordinary differential equations given in canonical state-variable form:

$$\frac{dx}{dt} = f_x(x,u), \quad x(t_0) = x_0, \quad t \geqslant t_0 \qquad (1)$$

where

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \qquad u = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}, \qquad f_x = \begin{bmatrix} f_{x1} \\ \vdots \\ f_{xn} \end{bmatrix}$$

For the case that n is very large, one is seldom interested in observing all the state variables. In such cases, one observes only a small subset of the state variables: i.e.,

$$q = g(x) \qquad\qquad (2)$$

where

$$q = \begin{bmatrix} q_1 \\ \vdots \\ q_p \end{bmatrix}, \qquad g = \begin{bmatrix} g_1 \\ \vdots \\ g_p \end{bmatrix}$$

and $p < n$. The function g is called the <u>aggregation function</u>.
The problem of aggregation is that of trying to find a simpler
model to generate the observed variables q than that provided
by equations (1) and (2). Ideally, one seeks an aggregated
model characterized by function f_q such that

$$\frac{dq}{dt} = f_q(q,u), \quad q(t_o) = g(x_o), \quad t \geqslant t_o \qquad (3)$$

where

$$f_q = \begin{bmatrix} f_{q1} \\ \vdots \\ f_{qp} \end{bmatrix}$$

In general there exists no f_q such that an aggregated
model can generate the observed variables of a disaggregated
model (If the state x is observable through output q, then q
cannot, in general, be generated by a system of dimension less
than n). In special cases where an f_q can be found such that
equation (3) is valid, the aggregated model is said to be <u>dy-
namically exact</u> to the disaggregated model with respect to q.
However, dynamic exactness is so rare in practical situations
that, practically, the problem of aggregation is that of find-
ing a function f_q that can be used to generate an approxima-
tion q_a to q:

$$\frac{dq_a}{dt} = f_q(q_a,u), \quad q_a(t_o) = g(x_o), \quad t \geqslant t_o \qquad (4)$$

where

$$q_a = \begin{bmatrix} q_{a1} \\ \vdots \\ q_{ap} \end{bmatrix}$$

Often, the variables of interest are so few in number compared to the dimension of the disaggregated model (e.g., one may be interested in only the average of all the state variables in a complex system having, say, 50,000 state variables) that there is little hope of finding any f_q to generate a reasonable approximation to q. In such cases, it is necessary to increase the dimension of the aggregated model. Toward this end the aggregation function is redefined:

$$q = \begin{bmatrix} q_c \\ q_v \end{bmatrix} = \begin{bmatrix} g_c(x) \\ g_v(x) \end{bmatrix}$$

where

$$q_c = \begin{bmatrix} q_{c1} \\ \vdots \\ q_{cr} \end{bmatrix} = \begin{bmatrix} g_{c1}(x) \\ \vdots \\ g_{cr}(x) \end{bmatrix} \quad , \quad q_v = \begin{bmatrix} q_{v,r+1} \\ \vdots \\ q_{vp} \end{bmatrix} = \begin{bmatrix} g_{v,r+1}(x) \\ \vdots \\ g_{vp}(x) \end{bmatrix}$$

where q_c represents the variables of interest and q_v represents the additional variables to be included in the aggregated model to increase model dimension for purposes of improving the approximation. Thus, function g_c is a fixed function defining the variables of interest and function g_v is a function to be selected in the most advantageous manner in designing the aggregated model.

In trading dynamic exactness for model simplicity, by accepting an approximation to q, a difficult problem arises. Namely, one must have a basis for comparing alternate approximations. Clearly, the effectiveness of an approximation is closely tied to the use that the aggregated model is to be put to. Thus, the criteria that might be used in evaluating aggregated models to be used for estimation and prediction could

be significantly different from the criteria used when the models are to be used for determining controls. The development of pertinent criteria for evaluating aggregated models is an essential part of the aggregation problem.

A QUANTITATIVE APPROACH TO AGGREGATION: STRUCTURAL SENSITIVITY

The approach taken to the aggregation problem is based on system structure. Specifically, system models are represented by graphs such as link-node structures (1), system diagrams (2), or signal-glow graphs (3, 4) in which certain points on the graphs represent system variables and the influence of one variable on another is denoted by the existence of a path from that variable to the other. Fundamental to this approach is the premise that a proposed aggregated model can be imbedded within a larger system defined by the disaggregated system (i.e., the funciton f_x) and the aggregation function (i.e., the functions g_c and g_v). Importantly, in order that the proposed aggregated model exactly generate the variables of interest q_c, it is necessary that additional variables, say x_Δ, which are functions of the state variables of the disaggregated model, be provided as special inputs to the aggregated model. These relations between the larger-system variables, x_Δ, and the proposed aggregated system variables, g_c and q_v, represent connections in the system graph. Perfect aggregation is achieved when the q_c generated by the aggregated model is totally insensitive to the existence of these connections.

With such insensitivity, all connections from the larger system can be literally cut and the aggregated model can be removed from the larger system. This sensitivity of a system's variables to the cutting of connecting links is called structural sensitivity. By introducing a gain parameter in such connecting links it is possible to relate structural sensitivities to the

well-defined parameter sensitivities (e.g., a link gain equal
to 1 implies the connection exists, and a link gain equal to 0
implies the connection is broken). The following example ill-
ustrates the proposed approach to aggregation.

Consider a continuous autonomous system that is well mod-
eled by

$$\frac{dx}{dt} = f_x(x)$$

(x is an n vector). We would like to design an aggregated model
of this system and we demand that the aggregated model generate
a specified set of outputs q_c defined by a fixed aggregation
function:

$$q_c = g_c(x)$$

(q_c is an r vector). However, although we wish to design an
r-th order aggregated model in which q_c is the state, we are
willing to increase the dimension of the aggregated model by
adding variables q_v to the state in the hope that the inclusion
of important dynamic modes in the aggregated model will lead to
a better approximation of q. The variables q_v are selected by
the designer as a function of the state variables:

$$q_v = g_v(x)$$

(q_v is a p vector). Thus, here we seek an aggregated model of
the form:

$$\frac{dq_c}{dt} = f_{qc}(q_c, q_v)$$

$$\frac{dq_v}{dt} = f_{qv}(q_c, q_v)$$

With proper care in selecting the aggregation functions g_c and g_v, so as to avoid algebraic dependencies among the elements of q_c and q_v, one can think of q_c and q_v as being a set or $r+p$ state variables in a new state description. Denoting the new state vector by $\hat{x}$, we have

$$\hat{x} = \begin{bmatrix} q_c \\ q_v \\ x_\Delta \end{bmatrix}$$

where x_Δ is an $n-p-r$ vector that is augmented to q_c and q_v to complete the state description. x_Δ is not unique and can be obtained by an appropriate transformation from the original state description.

$$x_\Delta = g_\Delta(x)$$

Thus, the transformation from x to $\hat{x}$ is given by

$$\hat{x} = \begin{bmatrix} g_c(x) \\ g_v(x) \\ g_\Delta(x) \end{bmatrix} = g_t(x)$$

where g_t is a transformation function and, as such, has an inverse: i.e.

$$x = g_t^{-1}(\hat{x})$$

Differentiating $\hat{x}$ with respect to time gives

$$\frac{d\hat{x}}{dt} = \begin{bmatrix} \nabla_x(g_c(x)) \\ \nabla_x(g_v(x)) \\ \nabla_x(g_\Delta(x)) \end{bmatrix} \frac{dx}{dt}$$

where $\nabla_x(g_c(x))$ is an $r \times n$ matrix such that each row of $\nabla_x(g_c(x))$ is the gradient, in the x space, of the corresponding element of $g_c(x)$. Thus for example, the i-th row of $\nabla_x(g_c(x))$ is

$$\nabla_x(g_{ci}(x)) = \left[\frac{\partial g_c}{\partial x_1} \frac{\partial g_c}{\partial x_2} \cdots \frac{\partial g_c}{\partial x_n}\right]$$

Similarly, $\nabla_x(g_v(x))$ is a pxn matrix and $\nabla_x(g_\nabla(x))$ is an $(n-p-r)$xn matrix. Since $\frac{dx}{dt} = f_x(x)$, we may write

$$\frac{d\hat{x}}{dt} = \begin{bmatrix} \nabla_x(g_c(x)) \\ \nabla_x(g_v(x)) \\ \nabla_x(g_\Delta(x)) \end{bmatrix} f_x(x)$$

And since $\hat{x} = g_t^{-1}(x)$, we obtain a set of transformed state equations for the original system:

$$\frac{d\hat{x}}{dt} = \begin{bmatrix} \nabla_x(g_c(g_t^{-1}(\hat{x}))) \\ \nabla_x(g_v(g_t^{-1}(\hat{x}))) \\ \nabla_x(g_\Delta(g_t^{-1}(\hat{x}))) \end{bmatrix} f_x(g_t^{-1}(\hat{x}))$$

or, equivalently

$$\frac{dq_c}{dt} = f_{qc}(q_c, q_v, x_\Delta)$$

$$\frac{dq_v}{dt} = f_{qv}(q_c, q_v, x_\Delta) \tag{5}$$

$$\frac{dx_\Delta}{dt} = f_{x\Delta}(q_c, q_v, x_\Delta)$$

where

$$f_{qc}(q_c, q_v, x_\Delta) = \nabla_x(g_c(g_t^{-1}(\hat{x}))) f_x(g_t^{-1}(\hat{x}))$$

$$f_{qv}(q_c, q_v, x_\Delta) = \nabla_x(g_v(g_t^{-1}(\hat{x}))) f_x(g_t^{-1}(\hat{x}))$$

$$f_{x\Delta}(q_c, q_v, x_\Delta) = \nabla_x(g_\Delta(g_t^{-1}(\hat{x}))) f_x(g_t^{-1}(\hat{x}))$$

Figure 1 shows the system diagram for this transformed system.
Clearly, if g_v and g_t can be selected so that q_c is completely
independent of x_Δ, then perfect aggregation is achieved. This
independence is equivalent to being able to cut the connection
marked with "X" without affecting q_c. Note that perfect aggre-
gation is achieved if $f_{qc}(q_c, q_v, q_\Delta)$ is insensitive to x_Δ. How-
ever, in terms of designing an aggregated model, this condition
may be much too strong. For example, suppose the system's oper-
ation is such that $\frac{dx_\Delta}{dt} \cong 0$ (i.e., $f_{x\Delta}(q_c, q_v, x_\Delta) \cong 0$) and $f_{x\Delta}$ is such
that $f_{x\Delta}(q_c, q_v, x_\Delta) = 0$ can be solved for x_Δ in terms of q_c and
q_v. In this case although $f_{qv}(q_c, q_v, x_\Delta)$ is a function of x_Δ,
the additional relation relating x_Δ to q_c and q_v makes perfect
aggregation possible.

In situations where no functions g_v and g_Δ can be found
that desensitize q_c to cutting the connections that bring x_Δ
to the f_{qc} and f_{qv} blocks, an approximation procedure is sug-
gested which is based on introducing a cutting parameter α.
Figure 2 shows the system diagram of equations (5) with such
a cutting parameter: $\alpha = 1$ gives the original system: $\alpha = 0$
cuts the connections. In this system, one may use the sensitiv-
ity of q_v to the cutting parameter α as an indicator of the effec-
tiveness of aggregation and proceed to look for the functions
g_v and g_Δ that minimize this sensitivity. It should be noted
that the sensitivity of q_c to α not only depends on the aggre-
gation function g_v but also on the set of state variables selec-
ted for x_Δ : i.e., on the selection function g_Δ.

The approach to the computation of the structural sensi-

tivities is straightforward. The outputs q_c are computed with $\alpha = 1$. For large systems, this computation could strain the capacity of the computer being used and, perhaps, prove to be impractical in certain cases. In such situations where the limits of the computer are being tested, it is essential that efficient computational algorithms be used (4, 5). By setting $\alpha = 0$, the smaller proposed aggregated system is separated from the larger system and the outputs of this smaller system, q_{ca}, are computed. The structural sensitivities are simply the differences:

$$\frac{\Delta q_c}{\Delta \alpha} = q_{ca} - q_c$$

Perfect aggregation (i.e., dynamic exactness) is achieved when the structural sensitivities are zero over the time interval of interest. However, given that dynamic exactness is generally not possible, one then seeks the functions g_V and g_Δ that in some way minimize the structural sensitivities. For example, a figure of merit can be defined in terms of the structural sensitivities. There are many possibilities for defining a figure of merit. Some examples are:

$$M_1 = \left| \frac{\Delta \dot{q}_c}{\Delta \alpha}(1) \right|$$

$$M_2 = \int_0^1 \left(\frac{\Delta q_c}{\Delta \alpha}(\tau) \right)^2 w(\tau) \, d\tau$$

$$M_3 = \int_0^1 \left| \frac{\Delta q_c}{\Delta \alpha}(\tau) \right| w(\tau) \, d\tau$$

$$M_4 = \int_0^1 \left| \frac{\Delta q_c}{\Delta \alpha}(\tau) \cdot \frac{\alpha}{q_c} \right| w(\tau) \, d\tau$$

Figure of merit M_1 stresses the importance of the final value of q_c; M_2 and M_3 consider the accuracy of q_c to be important over the entire time interval (the weighing function w allows the emphasis to vary over the time interval); and M_4 is in terms a normalized sensitivity for situations in which one is concerned with percent errors. Clearly, the figure of merit to be used in any instance depends on the system being modeled.

LINEAR SYSTEMS

Although it is quite unreasonable to expect that any tactical C^3 system could be realistically represented by a linear time-invariant model, it is nevertheless useful to look at the aggregation problem for this special case. Importantly, many large subsystems of tactical C^3 systems are well modeled by linear differential equations and some progress toward obtaining useful aggregated models can be made by aggregating individual subsystems separately. Further, some of the ideas set forth here are rather simply illustrated using linear systems as examples. However, it must be noted that the assumption of linearity gives rise to significant simplifications that do not exist for any other class of systems.

Consider the case that the system of interest is well modeled by tha set of linear differential equations, written in matrix form:

$$\frac{dx}{dt} = Ax \quad (= f_x(x))$$

where x is an n vector and A is an nxn matrix of constants with the element in the i-th row and the j-th column represented by a_{ij}. The variables of interest, which are to be outputs of the aggregated model, are linear combinations of the original set of state variables:

$$q_c = G_c x \quad (= g_c(x))$$

where q_c is an r vector (r<n) and G_c is an rxn matrix. Variables q_c are to be state variables of the aggregated model. Additional state variables q_v, where q_v is a p vector (p<n-r),

may be allowed in the aggregated model to improve accuracy:

$$q_v = G_v x \quad (= g_v(x))$$

where G_v is a pxn matrix. If q_c and q_v are considered to be state variables in a transformed coordinate system, an additional n-p-r state variables must be selected, also as a linear combinations of the original state variables, to complete the transformed state description:

$$x_\Delta = G_\Delta x \quad (= g_\Delta(x))$$

where x_Δ is an n-p-r vector and G_Δ is an (n-p-r)xn matrix. Thus, representing the new state description with the n vector $\hat{x}$, we have

$$\hat{x} = \begin{bmatrix} G_c \\ G_v \\ G_\Delta \end{bmatrix} x = Gx \quad (= g_t(x))$$

where G is an nxn nonsingular transformation matrix appropriately constructed from submatrices G_c, G_v, and G_Δ. Differentiating the transformation equation $\hat{x}=Gx$ with respect to t gives

$$\frac{d\hat{x}}{dt} = GAG^{-1}x = G_{qv\Delta}x$$

where

$$G_{qv\Delta} = GAG^{-1}$$

This can be written in expanded form as

$$\frac{dq_c}{dt} = G_{qq}q_c + G_{qv}q_v + G_{q\Delta}x_\Delta \quad (= f_{qc}(q_c,q_v,x_\Delta))$$

$$\frac{dq_v}{dt} = G_{vq}q_c + G_{vv}q_v + G_{v\Delta}x_\Delta \quad (= f_{qv}(q_c,q_v,x_\Delta)) \qquad (6)$$

$$\frac{dx_\Delta}{dt} = G_{\Delta q}q_c + G_{\Delta v}q_v + G_{\Delta\Delta}x_\Delta \quad (= f_{x\Delta}(q_c,q_v,x_\Delta))$$

where the matrix coefficients of q_c, q_v, x_Δ are the appropriate

partitions of $G_{cv\Delta}$. Figure 3 shows the system diagram corre-

sponding to equations (6) with the cutting parameter α included.

The objective, of course, is to find the transformation subma-

trices G_v and G_Δ that minimize the sensitivity of q_c to the cut-

ting parameter α.

A DIGITAL COMPUTER PROGRAM

A FORTRAN program was written which can be used in simplifying linear systems. The program determines the optimal simplified model (i.e., the coefficient matrices G_{cc} and G_{cv}) of specified dimension r+p from the original linear model of dimension n (n>r+p). The program minimizes a figure of merit which is defined in terms of the structural sensitivities and the time interval of interest. The program inputs are the original n-dimensional model (i.e., the coefficient matrix A) and the dimension of the simplified model to be developed. Powell's algorithm (6) is used to execute the minimization.

The program consists of two major parts (see Figure 4). One part consists of the algorithms required to compute the value of the figure of merit from the original model and the proposed aggregated model; the other part consists of the optimization algorithm. Figure 5 details the algorithm for the linear case. Note that the A and G_c matrices are specified as input and G_v and G_Δ are determined from the optimization process. In the program written, Powell's method (6) was used as the basis for the optimization algorithm; Appendix 1 details the optimizatic.. algorithm used to determine the optimal aggregated system (i.e., the matrices G_v and G_Δ) and the corresponding figure of merit. Appendix 2 gives the definitions of the important FORTRAN variables. A listing of the resulting FORTRAN program is given in the Appendix 3. The results of a sample run are given in Appendix 4.

CONCLUSIONS

The research done thus far on using structural sensitivities as a basis for a measure of effectiveness of aggregation seems promising. The FORTRAN program written, which determines aggregated models for linear systems, establishes the feasibility of the approach, at least for an important class of systems. Especially important, insofar as competitive systems is concerned, is that by using structural sensitivities in the design of an aggregated model, attention must be given to all excluded dynamic modes; it is simply not sufficient that the variables of interest appear to be reasonably approximated. In minimizing structural sensitivities it is virtually not possible to accidently overlook important system dynamics in the aggregated model.

Use of this FORTRAN program on a variety of examples makes it clear that additional research is required to answer important questions concerning both the effectiveness and the applicability of the approach. As a first step for future research, I suggest a complete rewriting of the aggregation program either in a language such as APL so as to enormously simplify the program or, if again in FORTRAN, on a system with efficient linear algebra software packages. Certainly, having an experienced programmer write the program in a well-documented modular form would be advisable. In addition, the optimization algorithm should be carefully studied for the purpose of improving its covergence properties. Other optimiza-

tion algorithm should also be explored to determine whether the efficiency of the search can be improved.

With an improved aggregation program, it will be possible to begin to categorize models in terms of whether or not they can be simplified using this approach. By characterizing the properties of models that cannot be aggregated using structural sensitivity measures, insights will be obtained that should allow us to either develop methods to extend the applicability of the approach or to better define the limits of the approach.

Fundamental questions have been raised which should be the subject of future research efforts:

1. To test the feasibility and effectiveness of the proposed approach to aggregation based on structural sensitivities by applying it to model well-known test systems from the literature (e.g., the examples used in references 7 and 8). This should provide a comparison of the proposed approach to aggregation to some of the existing aggregation methods in terms of the effort involved, the usefulness of the resulting model, and the generality of the method.

2. To identify a suitable subsystem of an actual Air Force tatical C^3 system to be the subject of a modeling and simulation effort based on the proposed approach to aggregation.

3. In designing aggregated models of large-scale systems, a minimization must be carried out with respect to a large parameter space. Such minimizations can be difficult,

especially when there are many local minima to contend with.
Attention should be given to trying to reduce the dimension
of the parameter space by' using the least number of parame-
ters possible in defining the transformation functions g_v
and g_Δ in this application.

4. By actually cutting the connecting links, the variables x_Δ
being fed to the proposed aggregated model are actually set
to zero. This is of no concern when dynamic exactness can
be achieved. However, when the aggregated model can only
generate an approximation to the variables of interest,
one should consider the possibility of introducing bias in-
puts to the aggregated model at the points where the links
have been cut.

5. Frequently, one may wish to constrain the form of the aggre-
gated model, even at the cost of having a deteriorated ag-
gregated model or one of higher dimension. For example, one
may require that the aggregated model be linear and time-
invariant so as to permit analytic studies of the model in-
stead of, or in addition to, simulation studies. Methods
for introducing this model constraint into the setting of
structural sensitivities should be studied. This possi-
bility was briefly considered and the simple ploy of re-
placing the first of equations (5) by the following equa-
tion seems promising:

$$\frac{dq_c}{dt} = f_q(q_c,q_v) + [f_{qc}(q_c,q_v,x_\Delta) - f_q(q_c,q_v)]$$

where function f_q characterizes the constrained aggregated model. However, further study is necessary to determine how variations in the constrained model parameters affect the dynamics of the sensitivity function.

6. Considerable effort should be directed toward developing rationales for various forms of figures of merits derived from structural sensitivities. Many possibilities come to mind, including integral forms (with and without weighting functions) and those based on final values, and the implications of each ought to be examined, particularly with respect to the relationship of the effectiveness of the aggregated model to the magnitude of the figure of merit.

7. Since the computations of sensitivities are so much simpler for static systems than for dynamic systems, the possibility of designing aggregated models by examining only the right-hand side of the canonical state equations should be carefully investigated.

8. Linear time-invariant systems should be studied as an important special case. Certainly, many important real systems are modeled as linear time-invariant systems. However, the fact that linear time-invariant systems yield to analysis can be quite helpful in developing valuable insights into the implications of structural sensitivities.

9. A system can be defined such that the sensitivities $\Delta q_c/\Delta\alpha$ appear as the system outputs. Using this system, the problem of aggregation can be cast as a control problem in which the sensitivities can be considered to be error sig-

nals to be driven to zero. Such an approach to the aggre-
gation problem should be studied. It seems likely that
certain aspects of control theory will prove useful here.

10. The proposed approach to aggregation should be studied
with respect to the zero-state response of systems to clas-
sical test inputs (e.g., unit steps, sinusoids, etc.). It
is clear that the aggregated models depend on the system's
initial state. Yet, in many systems it is unlikely that
certain system state variables will ever assume signifi-
cant values because of the large attenuations between the
input and the storage devices associated with those state
variables. In such cases, determining acceptable aggre-
gated models might be simplest by dealing only with zero-
state input-output responses. For the linear case, this
is equivalent to looking for aggregated transfer functions.

11. A study should be made on the controllability of systems
using controls derived from aggregated models. It seems
that such a study makes sense only if a weaker definition
of controllability is used so as to take into account the
extraordinary controls generally necessary before the neg-
lected dynamics modes can significantly affect the outputs.
Particular attention should be given to the role of the
extra state variables q_v which are included in the aggre-
gated model only for accuracy. It may be desirable to in-
clude some additional state variables for purposes of con-
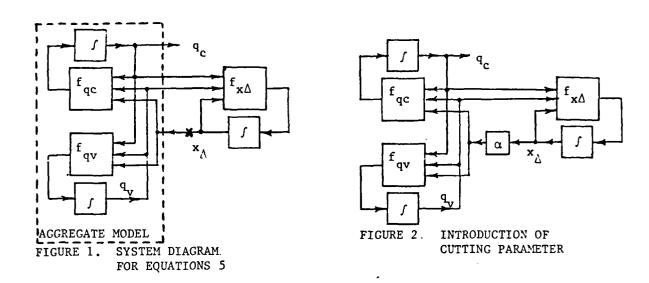trollability. For competitive systems, it is especially

important to determine the effect that an opponent can have
on controllability and to try to design an aggregated model
so as to minimize this effect.

12. A study should be made to extend the results to discrete-
    event systems.

13. A study should be made to examine the possibility of using
    structural sensitivity measures for decoupling subsystems.
    Such an application seems straightforward in that the coup-
    ling of subsystems can be represented graphically as links
    in the system graph. Then, all that is required is to find
    the transformations to minimize the sensitivity of the sub-
    system variables to the cutting of these links subject to
    the constraints that particular system variables be iden-
    tified with particular subsystems.

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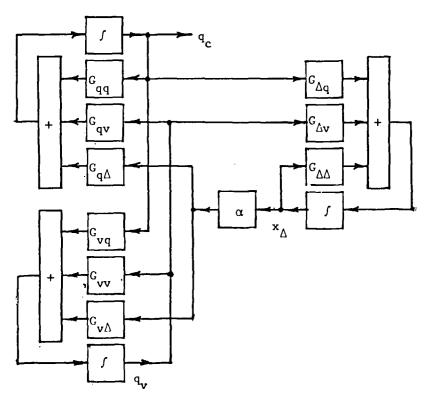
FIGURES

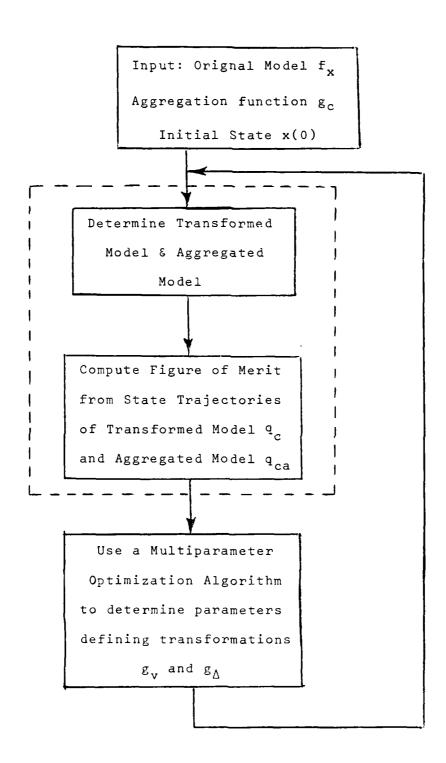AGGREGATE MODEL
FIGURE 1. SYSTEM DIAGRAM
FOR EQUATIONS 5

FIGURE 2. INTRODUCTION OF
CUTTING PARAMETER

FIGURE 3. LINEAR SYSTEM: SYSTEM DIAGRAM FOR EQUATIONS 6

```
┌─────────────────────────────┐
│  Input: Orignal Model f_x   │
│                             │
│  Aggregation function g_c   │
│                             │
│    Initial State x(0)       │
└─────────────────────────────┘
              │
              ▼
      ┌ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─ ─ ┐
      ┌─────────────────────────┐
      │   Determine Transformed │
      │    Model & Aggregated   │
      │         Model           │
      └─────────────────────────┘
      │           │             │
      │           ▼             │
      ┌─────────────────────────┐
      │  Compute Figure of Merit│
      │  from State Trajectories│
      │  of Transformed Model q_c│
      │  and Aggregated Model q_ca│
      └─────────────────────────┘
      └ ─ ─ ─ ─ ─ │ ─ ─ ─ ─ ─ ─ ┘
                  ▼
      ┌─────────────────────────┐
      │   Use a Multiparameter  │
      │  Optimization Algorithm │
      │  to determine parameters│
      │  defining transformations│
      │       g_v and g_Δ       │
      └─────────────────────────┘
```

Figure 4 Aggregation Algorithm

Input : $A, G_c, x(0)$

Transformation Matrix

$$G = \begin{bmatrix} G_c \\ G_v \\ G_\Delta \end{bmatrix}$$

$$G_{cv\Delta} = GAG^{-1}$$

Extract Aggregated System

$$\begin{bmatrix} G_{cc} & G_{cv} & \vdots & \bullet \\ G_{vc} & G_{vv} & \vdots & \bullet \\ \hdashline \bullet & \bullet & \vdots & \bullet \end{bmatrix} = G_{cv\Delta}$$

Initial Conditions

$$\hat{x}(0) = G_{cv\Delta}\, x(0)$$

$$\begin{bmatrix} q_c(0) \\ q_v(0) \\ \hdashline \bullet \end{bmatrix} = \hat{x}(0)$$

Optimization Subroutine Determines G_v & G_Δ (Powell-Fletcher Algorithm)

Transformed Model & Aggregated Model

$$\frac{d\hat{x}}{dt} = G_{cv\Delta}\, \hat{x}$$

$$\left\{ \frac{dq_{ca}}{dt} = G_{cc}\, q_{ca} + G_{cv}\, q_{va} \right.$$

$$\left. \frac{dq_{va}}{dt} = G_{vc}\, q_{va} + G_{vv}\, q_{va} \right\}$$

Figure of Merit

$$M = \int_0^{t_f} \left(\sum_{i=1}^{r} \left| \frac{q_{cai} - q_{ci}}{q_{ci}} \right| + \sum_{i=1}^{r} \left| \frac{q_{vai} - q_{vi}}{q_{vi}} \right| \right) dt$$

Figure 5 Aggregation Algorithm for Linear Models

APPENDIX 1

Powell's Multiparameter Optimization Method

Powell's method, which is perhaps the most powerful of
the direct search methods, has the property that it converges
to the exact optimum in a finite number of steps when the
function being optimized is a quadratic function. The algor-
ithm finds the maximum value of the function $f(x)$ and the m-
vector x giving that maximum. It is based on a series of one-
dimensional searches, each defined by an m-dimensional direc-
tion vector p. The method of selecting directions of the one
dimensional searches lies at the essence of the algorithm.

The algorithm proceeds as follows:

1) For each of the first m iterations, the direction vectors
   $p^{(1)}$, $p^{(2)}$, ..., $p^{(m)}$ are defined so as to step in each of
   the n coordinate directions.

2) α is the size of the step taken in direction p. The size
   of the steps in the first n iterations $(\alpha^{(1)}, \alpha^{(2)}, ...,$
   $\alpha^{(m)})$, however, is not predetermined. One moves in each
   direction until the maximum in that direction is determined.
   Thus for each step, a single variable search is conducted
   in one of the m directions, using any efficient single-
   variable search algorithm. In this program, a Fibonaci
   search was used.

3) Generate a new set of search direction vectors as follows

$$p^{(1)} = p^{(2)}$$

$$p^{(2)} = p^{(3)}$$

.
.
.

$$p^{(m-1)} = p^{(m)}$$

$$p^{(m)} = x^{(m)} - x^{(0)}$$

4) Define a new starting point $x^{(0)}$ as the maximum in the new $p^{(m)}$ direction

5) Repeat these steps with the new $x^{(0)}$ and the new directions $p^{(1)}, p^{(2)}, \ldots, p^{(m)}$

Powell has modified his algorithm to give better convergence when the initial guess of $x^{(0)}$ is a bad one.

1) Define $p^{(1)}, p^{(2)}, \ldots, p^{(m)}$ in the m coordinate directions. Start at $x^{(0)}$

2) For k=1,2,...m

$$x^{(k)} = x^{(k-1)} + \alpha^{(k)} p^{(k)}$$

where $\alpha^{(k)}$ is selected so as to maximize

$$f(x^{(k-1)} + \alpha^{(k)} p^{(1)}), \; \max_{\alpha^{(k)}} \left[f(x^{(k-1)} + \alpha^{(k)} p^{(k)}) \right] \equiv f(x^{(k)})$$

3) Find integer j, $1 \le j \le m$, so that $f(x^{(j)}) - f(x^{(j-1)})$ is a maximum and define $\Delta \equiv f(x^{(j)}) - f(x^{(j-1)})$

4) Compute

$$f_3 = f(x^{(0)} + 2(x^{(m)} - x^{(0)})) = f(2x^{(m)} - x^{(0)})$$

Define

$$f_1 = f(x^{(0)})$$

$$f_2 = f(x^{(m)})$$

5) If <u>either</u>

$$f_3 \geq f_1$$

<u>or</u>

$$(f_1-2f_2+f_3)(f_1-f_2-\Delta)^2 \geq \tfrac{1}{2}\Delta \cdot (f_1-f_3)^2$$

use old directions $p^{(1)}, p^{(2)}, \ldots, p^{(m)}$ and new $x^{(0)} = x$

for the next iteration.

6) If neither inequalities in (5) are true, define

$$p \equiv (x^{(m)}-x^{(0)}),$$

look for new $x^{(0)} = x^{(m)} + \alpha p$ where α minimizes $f(x^{(m)}+\alpha p)$

and define new direction vectors $p^{(1)}, p^{(2)}, \ldots, p^{(j-1)},$

$\ldots, p^{(m)}, p$

APPENDIX 2

Important FORTRAN Variables

A = an n^2 vector derived from the A matrix

$= a_{11} \ a_{21} \cdots a_{n1} \ a_{12} \ a_{22} \cdots a_{n2} \cdots a_{1n} \ a_{2n} \cdots a_{nn}$

GC = an $r \cdot n$ vector derived from the G_c matrix

$= g_{c11} \ g_{c21} \cdots g_{cr1} \cdots g_{c1n} \ g_{c2n} \cdots g_{crn}$

GV = a $p \cdot n$ vector derived from the the G_v matrix

$= g_{v11} \ g_{v21} \cdots g_{vp1} \cdots g_{v1n} \ g_{c2n} \cdots g_{cpn}$

GD = a $(n-r-p) \cdot n$ vector derived from the G_Δ matrix

$= g_{\Delta 11} \ g_{\Delta 21} \cdots g_{\Delta(n-r-p)1} \cdots g_{\Delta 1n} \ g_{\Delta 2n} \cdots g_{\Delta(n-r-p)n}$

Y = a $p \cdot n + (n-r-p) \cdot n$ vector derived from the G_v and the G_Δ matrices (Y is the vector to be determined in the minimization process)

$= [GV \vdots GD]$

G = an n^2 vector derived from the G_c, G_v, and G_Δ matrices

$= [GC \vdots GV \vdots GD]$

GAG = an n^2 vector derived from the $G_{cv\Delta}$ matrix

$(G_{cv\Delta} = GAG^{-1})$

X = an n vector representing the state of the original model

$(X = x)$

XHAT = an vector representing the state of the transformed model

$(XHAT = \hat{x})$

APPENDIX 3

FORTRAN Program

```
      COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
      COMMON E0,XX,H,JE,F0,F,F9,C9,F3,D,C8,N,J5,FGRAD
      COMMON/XXX/A(100),XS(10),GC(10),NN,RR,PP,DELT,FMERIT,TFINAL
      COMMON/GET1/GCC
      COMMON/TRAJEC/JOUT
      DIMENSION GCC(10)
      INTEGER RR,PP
      JE=0
      JOUT=0
C(150) INPUT CONVERGENCE DATA(730)
      CALL CONVDATA
C(350) INPUT INITIAL PARAMETERS(620)
      CALL INPPARAM
C  INPUT SYSTEM PARAMETERS
      CALL SYSPAR(A,XS,GC,RR,PP,DELT,TFINAL,NN)
C(230) OUTPUT INITIAL PARAMETERS AND CONVERGENCE DATA(840)
      CALL OUTPARAM
C  ESTIMATION OF PARAMETERS
C  *************************
C  *************************
C(370) COMPUTE INITIAL F(1820)
      CALL CMPINITF
C(390) INITIALIZE DIRECTION VECTOR P(1920)
      CALL INDIRVEC
C(410) STEP(2020) IN EACH DIRECTION P
410   CALL STEPP
C(430) CHECK FOR CONVERGENCE(2220)
      CALL CHEKCONV
C   C9=1 IMPLIES CONVERGENCE
      IF(C9.EQ.1) GO TO 590
C(450) FIND DIRECTION GIVING GREATEST INCREASE IN F(2420)
      CALL RAPIDF
C(470) COMPUTE TEST POINT F(2X(N)-X(0))(2530)
      CALL TESTPNT
C(490) CHECK WHETHER DIRECTION VECTORS ARE TO BE CHANGED(2600)
      CALL CHKDIRVK
C   C8=1 IMPLIES DIRECTION VECTORS ARE TO BE CHANGED
      IF(C8.EQ.1) GO TO 509
C(504) DON'T CHANGE DIRECTION VECTORS(2710)
      CALL DIRVECOK
C(507) REPEAT PROCESS WITH NEW STARTING VALUES
      GO TO 410
C(509) CHANGE DIRECTION VECTORS(2820)
509   CALL NEWDIREC
C(530) REVISE ESTIMATES OF PARAMETERS(2910)
      CALL REVEST
```

```
C(540) COMPUTE NEW DIRECTION VECTORS P(2960)
       CALL COMPNEWP
C(570)  REPEAT ESTIMATION PROCESS WITH NEW STARTING ESTIMATES
       GO TO 410
590    CONTINUE
       STOP
       END
730    SUBROUTINE CONVDATA
       COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
       COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
C  N=NUMBER OF PARAMETERS
C  EO=THE CONVERGENCE FACTOR FOR THE MAIN MINIMIZATION
C  CONVERGENCE IMPLIES  ABS(SUM(XN-XO)) < EO
C  H=THE CONVERGENCE FACTOR FOR THE ONE-DIMENSIONAL SEARCH
C  JE=THE NUMBER OF TIMES THE FUNCTION IS COMPUTED
       READ,N,EO,H,FGRAD
       RETURN
       END
620    SUBROUTINE INPPARAM
       COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
       COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
       READ,(S(I),I=1,N)
       DO 630 I=1,N
630    X(1,I)=S(I)
       RETURN
       END
840    SUBROUTINE OUTPARAM
       COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
       COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
       WRITE(01,842)N
842    FORMAT('THE NUMBER OF PARAMETERS IS',I5)
       WRITE(01,843)EO
843    FORMAT(' CONVERGENCE FACTOR FOR  MAIN MINIMIZATION :',F10.7)
       WRITE(01,844)H
844    FORMAT(' CONVERGENCE FACTOR FOR  ONE-DIMENSIONAL SEARCH :',F10.7)
       WRITE(01,845)FGRAD
845    FORMAT(' GRADIENT CUTOFF FOR ONE-DIM. SEARCH :',F10.7)
       WRITE(01,850)N
850    FORMAT('THE ',I5,' INITIAL PARAMETER VALUES ARE:')
       WRITE(01,855)(S(I),I=1,N)
855    FORMAT(10E12.4)
       RETURN
       END
1820   SUBROUTINE CMPINITF
       COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
       COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
       COMMON/XXX/A(100),XS(10),GC(10),NN,RR,PP,DELT,FMERIT,TFINAL
       INTEGER RR,PP
       DO 1850 I=1,N
1850   Y(I)=S(I)
       CALL FUNCTION
       FO=F
       RETURN
       END
```

```
4970    SUBROUTINE FUNCTION
        COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
        COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
        COMMON/XXX/A(100),XS(10),GC(10),NN,RR,PP,DELT,FMERIT,TFINAL
        INTEGER RR,PP
        JE=JE+1
        CALL MERIT(A,Y,GC,XS,NN,RR,PP,DELT,FMERIT,TFINAL)
        F=-FMERIT
4972    CONTINUE
        RETURN
        END
1920    SUBROUTINE INDIRVEC
        COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
        COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
        DO 1950 I=1,N
        DO 1950 J=1,N
1950    P(I,J)=0
        DO 2000 I=1,N
2000    P(I,I)=1
        RETURN
        END
2020    SUBROUTINE STEPP
        COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
        COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
        DO 2190 I=1,N
        DO 2120 J=1,N
        V(J)=P(I,J)
        IF(I.GT.1)GO TO 2110
        R(J)=S(J)
        GO TO 2120
2110    R(J)=X(I-1,J)
2120    CONTINUE
        CALL MAX
        FUNCT(I)=F
        DO 2180 J=1,N
2180    X(I,J)=R(J)+XX*V(J)
2190    CONTINUE
        F9=FUNCT(N)
        RETURN
        END
4180    SUBROUTINE MAX
        COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
        COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
        REAL M1,M2
        X4=0.1
        XO=0
4200    M1=XO-X4
        M2=XO+X4
        XX=XO
        CALL STEPFUNC
        QO=F
        XX=M1
        CALL STEPFUNC
        Q1=F
```

```
         XX=M2
         CALL STEPFUNC
         Q2=F
4330     IF(Q0.GT.Q1)GO TO 4400
         IF(Q0.GT.Q2)GO TO 4480
         WRITE(01,4300)
4300     FORMAT('MINIMUM DISCOVERED IN SUBROUTINE MAX')
         WRITE(6,4302)Q1,Q0,Q2
4302     FORMAT('Q1=',E12.4,'  Q0=',E12.4,'  Q2=',E12.4)
         IF(Q2.LT.Q1)GO TO 4350
         X0=M2
         GO TO 4200
4350     X0=M1
         GO TO 4200
         STOP
4400     IF(Q0.GT.Q2)GO TO 4550
4410     X4=2*X4
         M1=X0
         Q1=Q0
         X0=M2
         Q0=Q2
         M2=X0+X4
         FOLD=Q0
         XX=M2
         CALL STEPFUNC
         Q2=F
         IF(ABS(F-FOLD).GT.FGRAD)GO TO 4430
         GO TO 4570
4430     GO TO 4400
4480     X4=2*X4
         M2=X0
         Q2=Q0
         X0=M1
         Q0=Q1
         M1=X0-X4
         FOLD=Q0
         XX=M1
         CALL STEPFUNC
         Q1=F
         IF(ABS(F-FOLD).GT.FGRAD)GO TO 4485
         GO TO 4570
4485     GO TO 4330
4550     A=M1
         B=M2
         GO TO 4580
C    ******************************
C    ******* **********************
C    ONE-DIMENSIONAL SEARCH AN INTERVAL (A.B)
4580     ANEW=(A+B)/2-0.1*(B-A)
         BNEW=(A+B)/2+0.1*(B-A)
4582     FORMAT(2E15.5)
         XX=ANEW
         CALL STEPFUNC
         Q1NEW=F
         XX=BNEW
```

```
         CALL STEPFUNC
         Q2NEW=F
         IF(Q1NEW.GT.Q2NEW)GO TO 4585
         A=ANEW
         GO TO 4590
4585     B=BNEW
4590     IF((B-A).GT.H)GO TO 4580
         XX=(B+A)/2
         CALL STEPFUNC
4570     CONTINUE
         RETURN
         END
4760     SUBROUTINE STEPFUNC
         COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
         COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
         DO 4790 I=1,N
4790     Y(I)=R(I)+XX*V(I)
         CALL FUNCTION
4795     FORMAT(4E15.5)
         RETURN
         END
2220     SUBROUTINE CHEKCONV
         COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
         COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
         COMMON/GET1/GCC
         COMMON/TRAJEC/JOUT
         DIMENSION GCC(10)
         IF(ABS(FO-F9).GT.FGRAD)GO TO 2250
         WRITE(01,2240)
2240     FORMAT('FGRAD > ABS(FO-F9)')
         C9=1
         GO TO 2298
2250     SUM=0
         DO 2260 J=1,N
2260     SUM=SUM+ABS(X(N,J)-S(J))
         WRITE(01,2290)SUM,F
2290     FORMAT('SUM=',E12.4,'  F=',E12.4)
         WRITE(01,2294)N
2294     FORMAT('THE',I5,'  PARAMETER VALUES ARE:')
         WRITE(01,2295)(X(N,J),J=1,N)
2295     FORMAT(10E12.4)
         IF(SUM.GT.EO) GO TO 2325
         C9=1
2298     CALL SUMMARY
         JOUT=1
         CALL FUNCTION
         GO TO 2330
2325     C9=0
2330     RETURN
         END
2340     SUBROUTINE SUMMARY
         COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
         COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
         COMMON/GET1/GCC
```

```
      COMMON/XXX/A(100),XS(10),GC(10),NN,RR,PP,DELT,FMERIT,TFINAL
      DIMENSION GCC(10)
      INTEGER RR,PP
      WRITE(01,2350)F
2350  FORMAT('THE FUNCTION VALUE IS',E15.5)
      CALL GETOUT(GCC,Y,N,NN,RR,PP)
      WRITE(01,2390)JE
2390  FORMAT('THE FUNTION WAS COMPUTED   ',I10,'  TIMES.')
      RETURN
      END
2420  SUBROUTINE RAPIDF
      COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
      COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
      DI=FUNCT(1)-FC
      D=DI
      J5=1
      IF(N.EQ.1)GO TO 2515
      DO 2510 K=2,N
      DI=FUNCT(K)-FUNCT(K-1)
      IF(DI.LT.D)GO TO 2510
      D=DI
      J5=K
2510  CONTINUE
2515  CONTINUE
      RETURN
      END
2530  SUBROUTINE TESTPNT
      COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
      COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
      DO 2560 J=1,N
2560  Y(J)=2*X(N,J)-S(J)
      CALL FUNCTION
      F3=F
      RETURN
      END
2600  SUBROUTINE CHKDIRVK
      COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
      COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
      F5=(FO-2*F9+F3)*(FO-F9-D)**2
      F6=(1/2)*D*(FO-F3)**2
      IF(F3.LT.FO)GO TO 2670
      IF(F5.LT.F6)GO TO 2690
2670  C8=0
      GO TO 2700
2690  C8=1
2700  RETURN
      END
2710  SUBROUTINE DIRVECOK
      COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
      COMMON EO,XX,H,JE,FO,F,F9,C9,F3,D,C8,N,J5,FGRAD
      DO 2740 J=1,N
2740  S(J)=X(N,J)
      FO=F9
      RETURN
      END
```

```
2820    SUBROUTINE NEWDIREC
        COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
        COMMON E0,XX,H,JE,F0,F,F9,C9,F3,D,C8,N,J5,FGRAD
        DO 2860 J=1,N
        V(J)=X(N,J)-S(J)
2860    R(J)=X(N,J)
        CALL MAX
        F0=F
        RETURN
        END
2910    SUBROUTINE REVEST
        COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
        COMMON E0,XX,H,JE,F0,F,F9,C9,F3,D,C8,N,J5,FGRAD
        DO 2940 J=1,N
2940    S(J)=R(J)+XX*V(J)
        RETURN
        END
2960    SUBROUTINE COMPNEW
        COMMON S(10),Y(10),P(10,10),V(10),R(10),X(10,10),FUNCT(10)
        COMMON E0,XX,H,JE,F0,F,F9,C9,F3,D,C8,N,J5,FGRAD
        IF(J5.EQ.N)GO TO 3050
        J6=N-1
        DO 3010 I=J5,J6
        DO 3010 J=1,N
3010    P(I,J)=P(I+1,J)
        DO 3040 J=1,N
3040    P(N,J)=V(J)
3050    RETURN
        END
        SUBROUTINE GETOUT(GCC,Y,N,NN,RR,PP)
        INTEGER RR,PP,RP
        DIMENSION GCC(10),Y(10)
        WRITE(01,10)N
10      FORMAT('THE ',I5,'  FINAL PARAMETER VALUES ARE:')
        WRITE(01,12)(Y(I),I=1,N)
12      FORMAT(10E12.4)
        RP=RR+PP
        WRITE(01,15)RR,RR
15      FORMAT('THE ',I5,' X ',I5,'  AGGREGATED SYST. COEF. MATRIX:')
        DO 100 I=1,RP
        J1=I
        J2=(RP-1)*RP+I
        WRITE(01,200)(GCC(J),J=J1,J2,RP)
100     CONTINUE
200     FORMAT(10E12.4)
        RETURN
        END
        SUBROUTINE MERIT(A,Y,GC,XINIT,N,R,P,DELT,FMERIT,TFINAL)
        DIMENSION A(100),X(10),Y(10),GC(10),GD(10),G(100),GA(100)
        DIMENSION XINIT(10),XHAT(10),QCB(10),QCA(10)
        DIMENSION QC(10),LXX(10),MXX(10),GV(10),GAG(100),GCC(10)
        COMMON/GET1/GCC
        COMMON/TRAJEC/JOUT
        INTEGER R,P
```

```
C   EXTRACT GV FROM Y
      K=0
      IF(P.EQ.0)GO TO 1500
      DO 1001 I=1,P
      DO 1001 J=1,N
      K=K+1
1001  GV(K)=Y((I-1)*P+J)
1500  CONTINUE
C EXTRACT GD FROM Y
      NRP=N-R-P
      IF(NRP.EQ.0)GO TO 2010
      K=0
      DO 2000 I=1,NRP
      K=K+1
      GD((I-1)*N+1)=1
      NM1=N-1
      DO 2000 J=1,NM1
      K=K+1
2000  GD(K)=Y(P*N+(I-1)*NRP+J)
2010  CONTINUE
C   FORM G FROM GC, GV, AND GD
      K=0
      DO 6000 I=1,N
      DO 3000 J=1,R
      K=K+1
3000  G(K)=GC((I-1)*R+J)
      IF(P.EQ.0)GO TO 4500
      DO 4000 J=1,P
      K=K+1
4000  G(K)=GV((I-1)*P+J)
4500  CONTINUE
      IF(NRP.EQ.0)GO TO 6000
      DO 5000 J=1,NRP
      K=K+1
5000  G(K)=GD((I-1)*NRP+J)
6000  CONTINUE
C   COMPUTE XHAT(0)
      CALL GMPRD(G,XINIT,XHAT,N,N,1)
C   EXTRACT QCA(0) FROM XHAT(0)
      K=0
      DO 6555 I=1,R
      K=K+1
6555  QCA(K)=XHAT(K)
      T=0
      IF(JOUT.EQ.0)GO TO 6666
      WRITE(01,9550)T,(QCA(I).QCA(I),I=1,R)
6666  CONTINUE
C OBTAIN GAG
      CALL GMPRD(G.A,GA,N,N,N)
      CALL MINV(G.N.DXX,LXX,MXX)
      CALL GMPRD(GA.G,GAG,N,N,N)
C   EXTRACT GCC FROM GAG
      K=0
      KRP=R+P
```

```
              DO 7000 I=1,KRP
              DO 7000 J=1,KRP
              K=K+1
7000          GCC(K)=GAG((I-1)*N+J)
              FMERIT=0
              T=0
8000          IF(T.GT.TFINAL)GO TO 9999
C   XHAT(T+DELT)=XHAT(T)+DELT*(GAG*XHAT(T))
              CALL GMPRD(GAG,XHAT,X,N,N,1)
              DO 9000 I=1,N
9000          X(I)=DELT*X(I)
              CALL GMADD(XHAT,X,XHAT,N,1)
C   EXTRACT QC FROM XHAT
              K=0
              DO 9300 I=1,R
              K=K+1
9300          QC(K)=XHAT(K)
C   QCA(T+DELT)=QCA(T)+DELT*(GCC*QCA(T))
              CALL GMPRD(GCC,QCA,QCB,R,R,1)
              DO 9500  I=1,R
9500          QCB(I)=DELT*QCB(I)
              CALL GMADD(QCA,QCB,QCA,R,1)
C   COMPUTE FMERIT
              DO 9600 I=1,R
              FMERIT=FMERIT+DELT*ABS((QCA(I)-QC(I))/QC(I))
9600          CONTINUE
              IF(JOUT.EQ.0)GO TO 9555
              TD=T+DELT
              WRITE(01,9550)TD,(QC(I),QCA(I),I=1,R)
9550          FORMAT(F10.6,10E12.4)
9555          CONTINUE
              T=T+DELT
              GO TO 8000
9999          CONTINUE
              RETURN
              END
              SUBROUTINE SYSPAR(A,X,GC,R,P,DELT,TFINAL,N)
              DIMENSION A(100),X(10),GC(10)
              INTEGER R,P
              READ, N,R,P,DELT,TFINAL
              WRITE(01,39)N
39            FORMAT('DIMENSION OF ORIGINAL LINEAR SYSTEM:',I5)
              NN=N*N
              WRITE(01,79)R
79            FORMAT('NUMBER OF STATE VARIABLES OF INTEREST:  ',I5)
              WRITE(01,78)P
78            FORMAT('NUMBER OF EXTRA STATE VARIABLES:  ',I5)
              WRITE(01,77)DELT,TFINAL
77            FORMAT('INTEGRATION INTERVAL=',F10.5,'  TIME INTERVAL=',F10.5)
              WRITE(01,75)
75            FORMAT('ORIGINAL SYSTEM A-MATRIX:')
              DO 70 I=1,N
              J1=I
              J2=(N-1)*N+I
```

```
          READ,(A(J),J=J1,J2,N)
70        WRITE(01,76)(A(J),J=J1,J2,N)
76        FORMAT(10F12.4)
          READ,(X(I),I=1,N)
          WRITE(01,55)
55        FORMAT('THE INITIAL STATE IS:')
          WRITE(01,59)(X(I),I=1,N)
59        FORMAT(10E12.4)
          WRITE(01,65)R,N
65        FORMAT('THE ',I5,' X ',I5,' AGGREGATION MATRIX:')
          DO 60 I=1,R
          J1=I
          J2=(N-1)*R+I
          READ,(GC(J),J=J1,J2,R)
60        WRITE(01,76)(GC(J),J=J1,J2,R)
          RETURN
          END
/INC GMPRD,MINV,GMADD
/DATA
2,.01,.01,.000001
0,0,0
3,2,0,.05,1
0,1,0
-257.9047,-10,0
1,1,-5
1,1,1
1,.00001,.00001
.00001,1,.00001
```

APPENDIX 4

A Sample Run

```
DIMENSION OF ORIGINAL LINEAR SYSTEM:     3
NUMBER OF STATE VARIABLES OF INTEREST:      2
NUMBER OF EXTRA STATE VARIABLES:       0
INTEGRATION INTERVAL=   0.01000  TIME INTERVAL=    1.00000
ORIGINAL SYSTEM A-MATRIX:
        0.0000        1.0000        0.0000
     -257.9048      -10.0000        0.0000
        1.0000        1.0000       -5.0000
THE INITIAL STATE IS:
 -0.1000E 01 -0.1000E 01 -0.1000E 01
THE      2 X      3 AGGREGATION MATRIX:
        1.0000        1.0000        1.0000
        1.0000        0.0000        0.0000
THE NUMBER OF PARAMETERS IS     2
 CONVERGENCE FACTOR FOR  MAIN MINIMIZATION : 0.0100000
 CONVERGENCE FACTOR FOR  ONE-DIMENSIONAL SEARCH : 0.0100000
 GRADIENT CUTOFF FOR ONE-DIM. SEARCH : 0.0000010
THE      2 INITIAL PARAMETER VALUES ARE:
 0.0000E 00  0.0000E 00
SUM=  0.1071E 01  F= -0.2782E 00
THE     2  PARAMETER VALUES ARE:
 0.2383E-01 -0.1047E 01
SUM=  0.1332E 00  F= -0.5726E-01
THE     2  PARAMETER VALUES ARE:
 0.1567E-01 -0.9223E 00
SUM=  0.9421E-02  F= -0.3173E-01
THE     2  PARAMETER VALUES ARE:
 0.1692E-01 -0.9304E 00
THE FUNCTION VALUE IS   -0.31729E-01
THE      2 FINAL PARAMETER VALUES ARE:
 0.1692E-01 -0.9304E 00
THE      2 X      2  AGGREGATED SYST. COEF. MATRIX:
 -0.7946E 01 -0.2458E 03
  0.9821E 00 -0.2038E 01
THE FUNTION WAS COMPUTED          116  TIMES.
```

T (TIME UNITS)	Q1(T) (ORIGINAL SYSTEM VARIABLE)	QA1(T) (AGGRE- GATED SYSTEM VARIABLE)	Q2(T) (ORIGINAL SYSTEM VARIABLE)	QA2(T) (AGGREGATED SYSTEM VARIABLE)
0.000000	-0.3000E 01	-0.3000E 01	-0.1000E 01	-0.1000E 01
0.010000	-0.3010E 00	-0.3037E 00	-0.1010E 01	-0.1009E 01
0.020000	0.2208E 01	0.2201E 01	-0.9932E 00	-0.9915E 00
0.030000	0.4476E 01	0.4463E 01	-0.9520E 00	-0.9497E 00
0.040000	0.6463E 01	0.6442E 01	-0.8894E 00	-0.8865E 00
0.050000	0.8137E 01	0.8109E 01	-0.8084E 00	-0.8052E 00
0.060000	0.9479E 01	0.9444E 01	-0.7127E 00	-0.7091E 00
0.070000	0.1048E 02	0.1044E 02	-0.6056E 00	-0.6019E 00
0.080000	0.1114E 02	0.1109E 02	-0.4909E 00	-0.4871E 00
0.090000	0.1146E 02	0.1140E 02	-0.3720E 00	-0.3683E 00
0.100000	0.1146E 02	0.1140E 02	-0.2523E 00	-0.2488E 00
0.110000	0.1118E 02	0.1111E 02	-0.1350E 00	-0.1318E 00
0.120000	0.1062E 02	0.1055E 02	-0.2299E-01	-0.2000E-01

0.130000	0.9834E 01	0.9760E 01	0.8135E-01	0.8402E-01
0.140000	0.8854E 01	0.8778E 01	0.1758E 00	0.1782E 00
0.150000	0.7720E 01	0.7642E 01	0.2588E 00	0.2607E 00
0.160000	0.6472E 01	0.6394E 01	0.3289E 00	0.3305E 00
0.170000	0.5150E 01	0.5074E 01	0.3853E 00	0.3866E 00
0.180000	0.3795E 01	0.3721E 01	0.4276E 00	0.4285E 00
0.190000	0.2444E 01	0.2372E 01	0.4558E 00	0.4563E 00
0.200000	0.1130E 01	0.1062E 01	0.4701E 00	0.4703E 00
0.210000	-0.1136E 00	-0.1788E 00	0.4712E 00	0.4712E 00
0.220000	-0.1262E 01	-0.1323E 01	0.4601E 00	0.4598E 00
0.230000	-0.2291E 01	-0.2348E 01	0.4379E 00	0.4374E 00
0.240000	-0.3184E 01	-0.3236E 01	0.4061E 00	0.4055E 00
0.250000	-0.3928E 01	-0.3976E 01	0.3662E 00	0.3654E 00
0.260000	-0.4515E 01	-0.4558E 01	0.3198E 00	0.3189E 00
0.270000	-0.4941E 01	-0.4980E 01	0.2685E 00	0.2677E 00
0.280000	-0.5207E 01	-0.5242E 01	0.2142E 00	0.2133E 00
0.290000	-0.5319E 01	-0.5350E 01	0.1584E 00	0.1575E 00
0.300000	-0.5285E 01	-0.5312E 01	0.1026E 00	0.1017E 00
0.310000	-0.5116E 01	-0.5139E 01	0.4834E-01	0.4748E-01
0.320000	-0.4828E 01	-0.4848E 01	-0.3158E-02	-0.3961E-02
0.330000	-0.4436E 01	-0.4453E 01	-0.5075E-01	-0.5149E-01
0.340000	-0.3958E 01	-0.3972E 01	-0.9350E-01	-0.9418E-01
0.350000	-0.3413E 01	-0.3425E 01	-0.1307E 00	-0.1313E 00
0.360000	-0.2820E 01	-0.2830E 01	-0.1617E 00	-0.1622E 00
0.370000	-0.2198E 01	-0.2207E 01	-0.1863E 00	-0.1867E 00
0.380000	-0.1565E 01	-0.1572E 01	-0.2042E 00	-0.2046E 00
0.390000	-0.9381E 00	-0.9446E 00	-0.2156E 00	-0.2159E 00
0.400000	-0.3334E 00	-0.3389E 00	-0.2205E 00	-0.2208E 00
0.410000	0.2354E 00	0.2306E 00	-0.2194E 00	-0.2196E 00
0.420000	0.7561E 00	0.7520E 00	-0.2127E 00	-0.2128E 00
0.430000	0.1219E 01	0.1215E 01	-0.2010E 00	-0.2011E 00
0.440000	0.1617E 01	0.1613E 01	-0.1850E 00	-0.1851E 00
0.450000	0.1943E 01	0.1940E 01	-0.1654E 00	-0.1655E 00
0.460000	0.2195E 01	0.2192E 01	-0.1430E 00	-0.1430E 00
0.470000	0.2373E 01	0.2370E 01	-0.1186E 00	-0.1186E 00
0.480000	0.2476E 01	0.2473E 01	-0.9295E-01	-0.9291E-01
0.490000	0.2508E 01	0.2505E 01	-0.6679E-01	-0.6673E-01
0.500000	0.2473E 01	0.2470E 01	-0.4085E-01	-0.4077E-01
0.510000	0.2377E 01	0.2374E 01	-0.1578E-01	-0.1568E-01
0.520000	0.2227E 01	0.2224E 01	0.7836E-02	0.7955E-02
0.530000	0.2031E 01	0.2027E 01	0.2950E-01	0.2963E-01
0.540000	0.1797E 01	0.1793E 01	0.4879E-01	0.4894E-01
0.550000	0.1535E 01	0.1531E 01	0.6540E-01	0.6556E-01
0.560000	0.1252E 01	0.1248E 01	0.7908E-01	0.7925E-01
0.570000	0.9583E 00	0.9539E 00	0.8971E-01	0.8990E-01
0.580000	0.6619E 00	0.6572E 00	0.9724E-01	0.9743E-01
0.589999	0.3704E 00	0.3655E 00	0.1017E 00	0.1019E 00
0.599999	0.9120E-01	0.8596E-01	0.1032E 00	0.1034E 00
0.609999	-0.1695E 00	-0.1751E 00	0.1019E 00	0.1022E 00
0.619999	-0.4065E 00	-0.4122E 00	0.9814E-01	0.9835E-01
0.629999	-0.6152E 00	-0.6212E 00	0.9209E-01	0.9230E-01
0.639999	-0.7925E 00	-0.7987E 00	0.8411E-01	0.8432E-01
0.649999	-0.9361E 00	-0.9425E 00	0.7456E-01	0.7475E-01
0.659999	-0.1045E 01	-0.1051E 01	0.6379E-01	0.6397E-01

```
0.669999 -0.1118E 01 -0.1125E 01  0.5218E-01  0.5234E-01
0.679999 -0.1158E 01 -0.1164E 01  0.4008E-01  0.4023E-01
0.689999 -0.1164E 01 -0.1171E 01  0.2785E-01  0.2797E-01
0.699999 -0.1140E 01 -0.1146E 01  0.1580E-01  0.1591E-01
0.709999 -0.1088E 01 -0.1094E 01  0.4244E-02  0.4324E-02
0.719999 -0.1012E 01 -0.1018E 01 -0.6565E-02 -0.6512E-02
0.729999 -0.9152E 00 -0.9211E 00 -0.1640E-01 -0.1638E-01
0.739999 -0.8020E 00 -0.8077E 00 -0.2509E-01 -0.2509E-01
0.749999 -0.6766E 00 -0.6818E 00 -0.3248E-01 -0.3251E-01
0.759999 -0.5429E 00 -0.5477E 00 -0.3849E-01 -0.3855E-01
0.769999 -0.4051E 00 -0.4095E 00 -0.4306E-01 -0.4314E-01
0.779999 -0.2670E 00 -0.2709E 00 -0.4618E-01 -0.4628E-01
0.789999 -0.1322E 00 -0.1356E 00 -0.4787E-01 -0.4800E-01
0.799999 -0.4037E-02 -0.6846E-02 -0.4821E-01 -0.4835E-01
0.809999  0.1148E 00  0.1125E 00 -0.4728E-01 -0.4744E-01
0.819999  0.2219E 00  0.2202E 00 -0.4519E-01 -0.4536E-01
0.829999  0.3154E 00  0.3142E 00 -0.4210E-01 -0.4228E-01
0.839999  0.3938E 00  0.3931E 00 -0.3815E-01 -0.3833E-01
0.849999  0.4563E 00  0.4561E 00 -0.3351E-01 -0.3369E-01
0.859999  0.5024E 00  0.5027E 00 -0.2835E-01 -0.2852E-01
0.869999  0.5322E 00  0.5328E 00 -0.2284E-01 -0.2300E-01
0.879999  0.5461E 00  0.5470E 00 -0.1715E-01 -0.1730E-01
0.889999  0.5448E 00  0.5461E 00 -0.1144E-01 -0.1158E-01
0.899999  0.5297E 00  0.5311E 00 -0.5863E-02 -0.5977E-02
0.909999  0.5020E 00  0.5036E 00 -0.5458E-03 -0.6385E-03
0.919999  0.4635E 00  0.4652E 00  0.4391E-02  0.4321E-02
0.929999  0.4159E 00  0.4176E 00  0.8849E-02  0.8801E-02
0.939999  0.3611E 00  0.3628E 00  0.1275E-01  0.1272E-01
0.949999  0.3011E 00  0.3027E 00  0.1603E-01  0.1603E-01
0.959999  0.2378E 00  0.2392E 00  0.1865E-01  0.1867E-01
0.969999  0.1731E 00  0.1743E 00  0.2060E-01  0.2064E-01
0.979999  0.1087E 00  0.1097E 00  0.2187E-01  0.2193E-01
0.989999  0.4634E-01  0.4710E-01  0.2248E-01  0.2256E-01
0.999999 -0.1258E-01 -0.1210E-01  0.2247E-01  0.2257E-01
1.009998 -0.6679E-01 -0.6661E-01  0.2188E-01  0.2199E-01
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This research effort is directed at determining the feasibility of using structural-sensitivity measures as the basis for simplifying large mathematical models of dynamic systems (e.g., models of tactical command, control, and communication systems). Toward this end, a FORTRAN program was written which can be used to simplify models characterized by sets of linear differential equations. Specifically, the program determines the optimal simplified model (i.e., the set of coefficients characterizing the set of linear differential equations) of specified dimension corresponding to the original linear model of higher dimension.

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20. Abstract cont.

The algorithm, on which the FORTRAN program is based, minimizes an objective
function which is defined in terms of the structural sensitivities of the state
variables to be preserved in the simplified model. The program inputs are the
set of coefficients which define (1) the linear differential equations repre-
senting the original model, (2) the dimension of the simplified model to be
determined, and (3) the set of parameters defining the objective function to be
minimized. The program outputs are the set of coefficients which define the
optimal simplified model and the value of the objective function corresponding
to the optimal simplified model.